EDP308: STATISTICAL LITERACY

The University of Texas at Austin, Fall 2020

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Overview

- The Uncertainty of Estimates
- Confidence Intervals
  - Point Estimate
  - Level of Desired Confidence
  - Margin of Error (MOE)
- Comparing Sample to Sampling Distributions
- Exact 68% (90%), 95%, 99% Rule
  - $z = 1.96$
- An Ideal World Example
- Uncertainty and 95% Confidence

Zoom Out – What's the Goal?

What is our goal when we take a sample and calculate a statistic like the mean?

Zoom Out – What's the Goal?

We want the truth!

We are sampling and calculating in an attempt to capture the truth, in this case true μ

More or Less Truths?

- Imagine you hear in the news:
  - "Purple candidate is expected to have 52% of the votes and is projected to win by 2% points in the up coming election. Green candidate is expected to have 48% of the vote."
- Who's going to win? Purple or Green?
- What if I told you the poll is accurate to within 7 points?
  - Purple is expected to have between 45-59% of the votes
  - Green is expected to have between 41-55% of the votes
- Who's going to win? Do we know?

More or Less

- The inaccuracy of polls is to be expected because we are sampling a subset, but research is vulnerable to the same inaccuracy issues.
- Every time you hear something like, “The average American watches 5 hours of TV per day.”
  - There is always some error in such statistics because statistics are derived from a sample
- A more accurate way to report the average amount of TV watching is to report a confidence interval, a range
  - “The average American watches between 4 to 6 hours of TV per day.”

Confidence Intervals

- Point Estimates
- Margin of Error
- Confidence Interval

Jellybeans!

- First, guess how many jellybeans are in the jar.
  - Ex. 100
- Second, a ± value
  - 100 ± 20
- Next, give me a range for your guess
  - Ex. 80-120
- Lastly, tell me how confident you are in your estimate
  - Ex. 95%, 80%, etc.

Give me an estimate…

- Point Estimate: This is what you just gave me as your jellybean guess
- Margin of Error (MOE): This is the ± value
- Confidence Interval (CI): This is range you calculated after you took your point estimate and ± the MOE
- Confidence Level: This is how confident you are that the true # of jellybeans falls within you CI

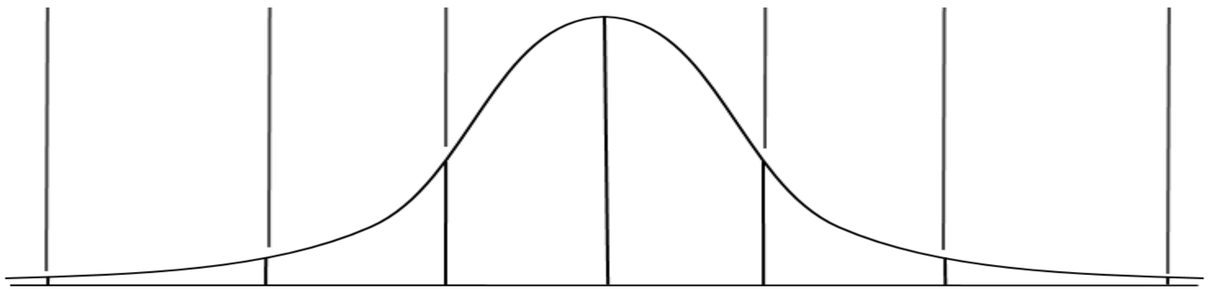
Confidence Intervals

- Confidence intervals are in the raw number language of the original scale
  - Ex. Jelly beans, height, exam scores, etc.

The number of jelly beans in the jar is 300

The number of jelly beans in the jar is between 250-350

95% CI [250, 350]



$\bar{x} = 300$

Lower Guess = 250

Upper Guess = 350

Confidence Levels

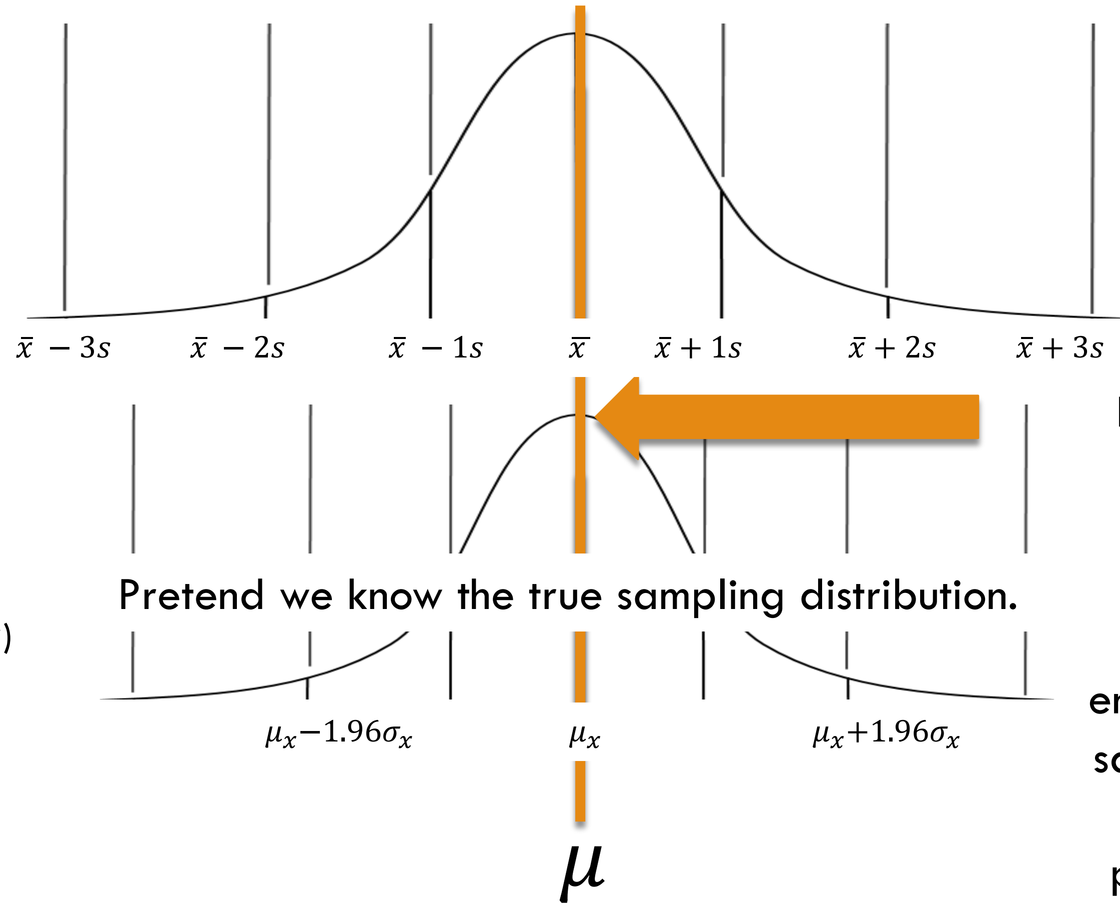
- Confidence Intervals describe the uncertainty and errors associated with taking a sample from the population
  - Point estimate $\pm$ Margin of Error
- We can't really know the absolute truth of a population when taking a sample (due to error).
  - Though our point estimate may be wrong, we can be confident that the extra cushion (the margin of error) around our estimate captures the truth, the true μ
    - This is our goal… To capture the truth.

Sample Compared to Sampling

Data Distribution:
$\bar{x}$, s
Distribution of the data from a ONE sample taken from the population

Sampling Distribution:
$\mu_{\bar{x}}$, $\sigma_{\bar{x}}$
Distribution of the possible statistics ($\bar{x}$, s) of many samples

Population's True μ



Perfect Match Scenario

Remember, with proper, large enough samples the sampling mean will equal true population mean.

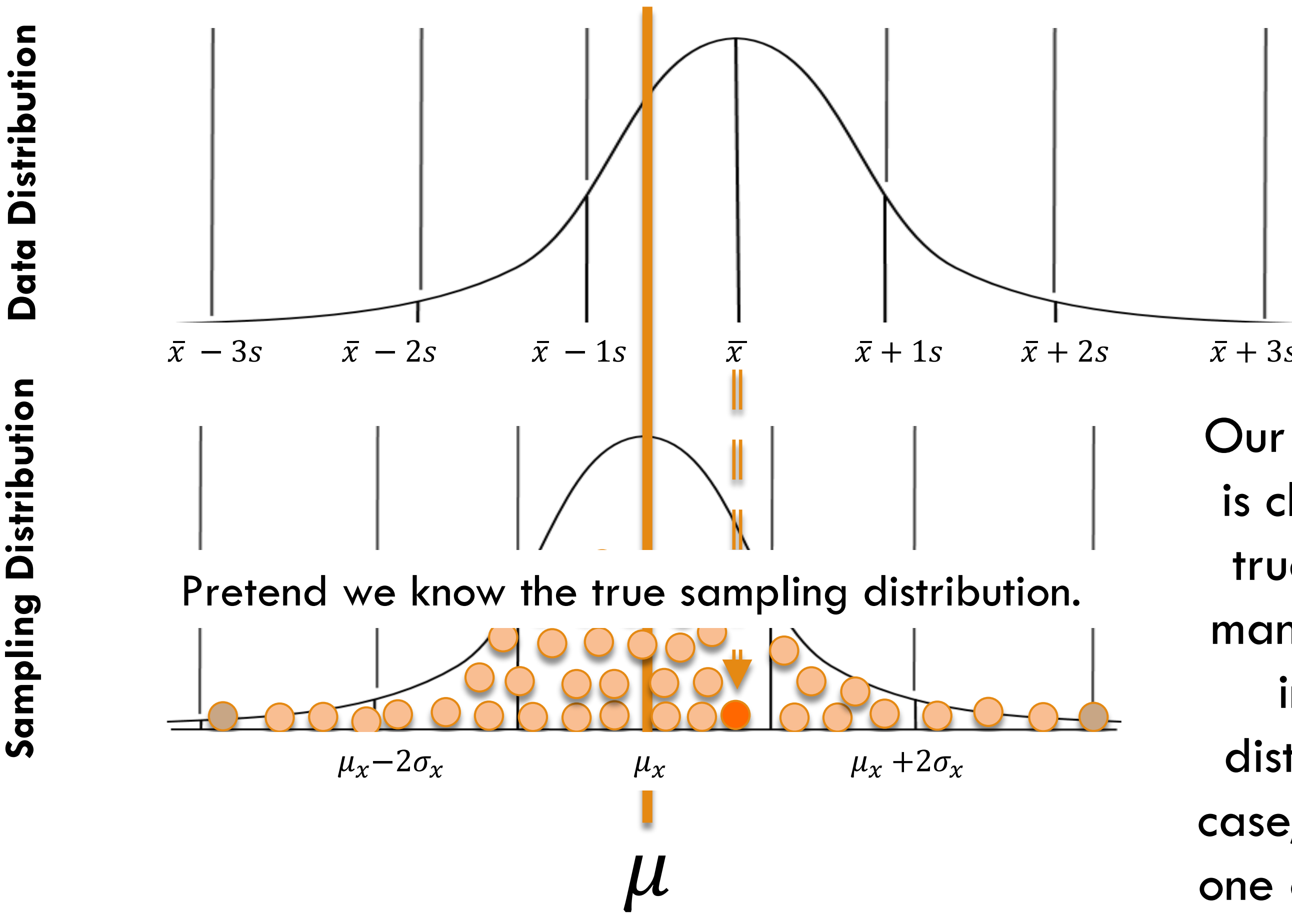
Ideal situation where out Point Estimate is equal to μ

Not very likely…

Quick Reminder

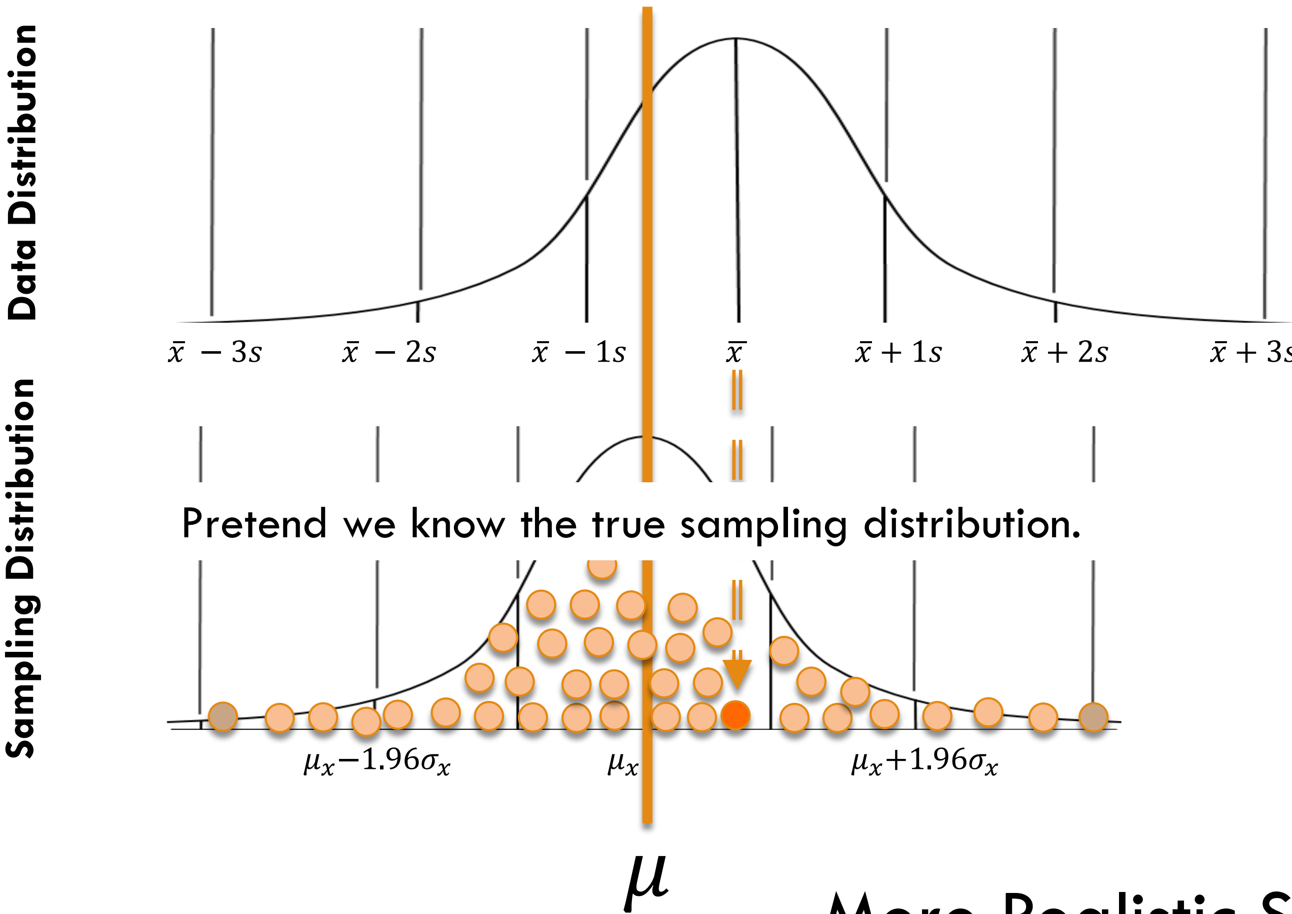
- Sampling Distributions are theoretical. Though they represent the distribution of a bunch of sample means, we do not actually go out, collect, and calculate 100s of sample means.
  - **Rather, we use our ONE sample and create a theoretical sampling distribution based on our one sample's mean, variance, and sample size.**
- While we are being introduced to these new topics, we are going to *pretend* as though we know the truth before moving on.

Sample Compared to Sampling



Our one sample mean $\bar{x}$ is close but not exactly true μ. It is one of the many sample means ($\bar{x}$) in the SAMPLING distribution ($\mu_{\bar{x}}$). In this case, our sample mean is one of the slightly higher estimates in sampling distribution.

Sample Compared to Sampling

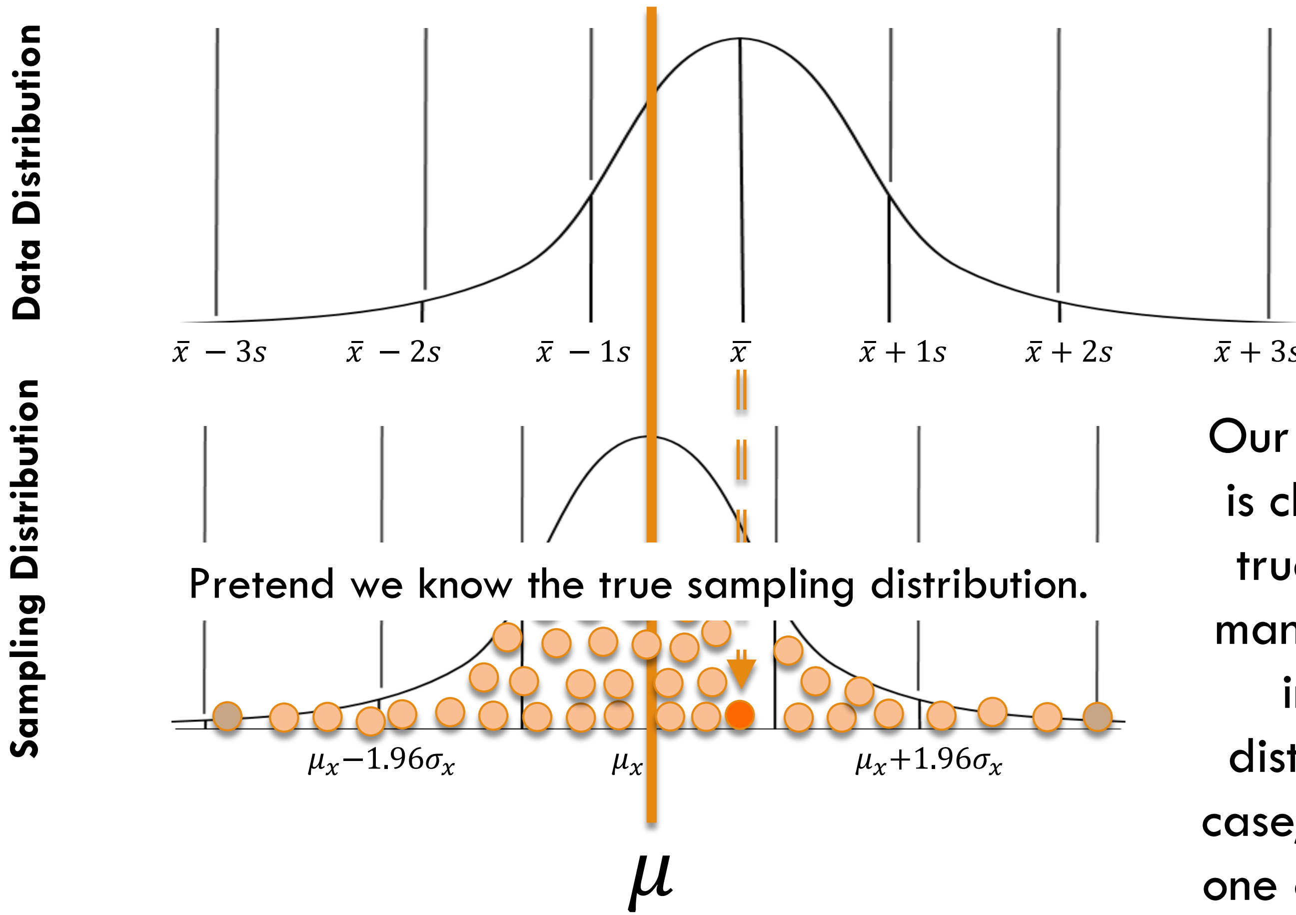


Notice how our one sample mean $\bar{x}$ is close but not exactly true μ

More Realistic Scenario

Our SAMPLE mean $\bar{x}$ won't be exactly μ

Sample Compared to Sampling



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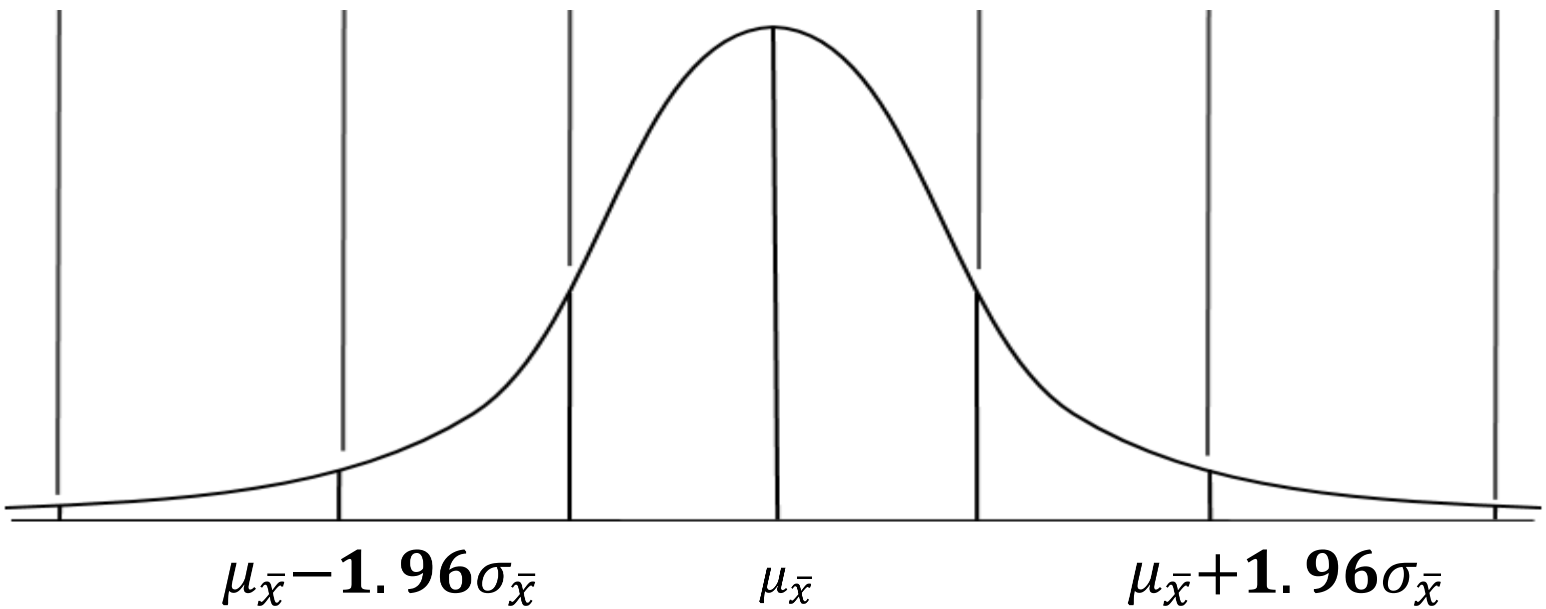
Wait, what’s with the 1.96??



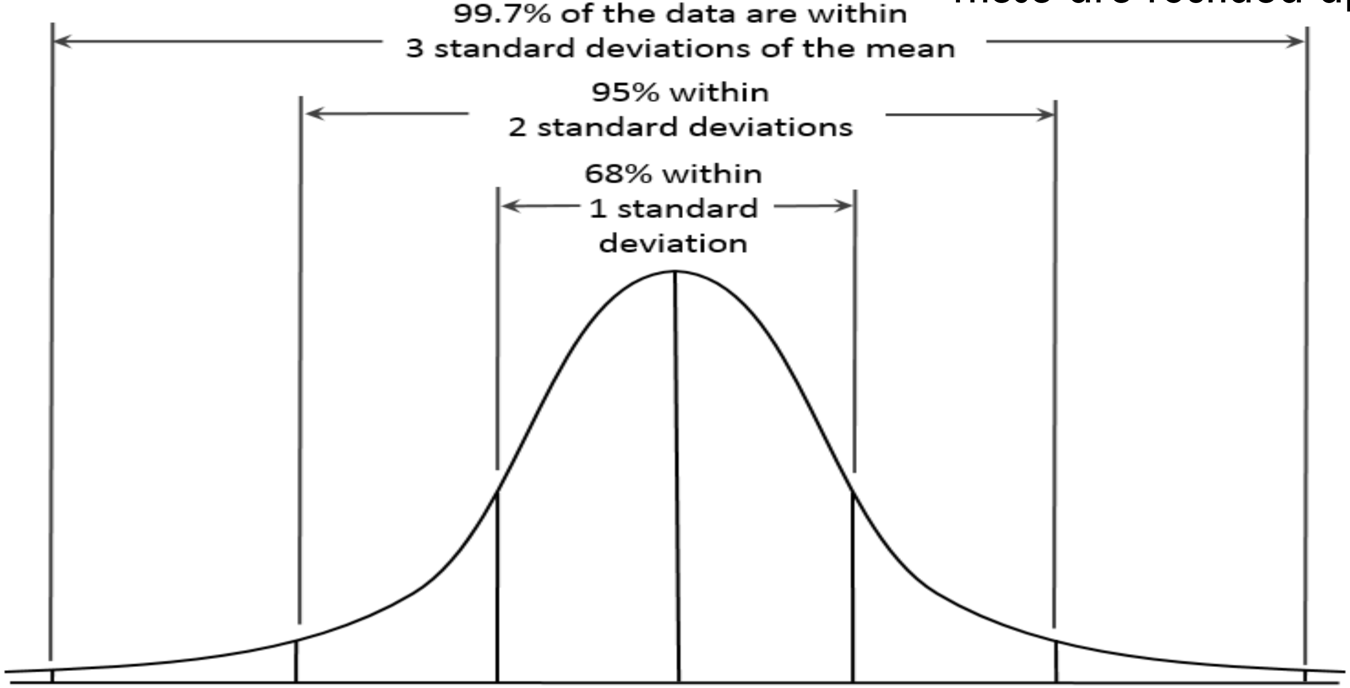
Remember the 68%, 95% 99% rules? For normally distributed data (like sampling distributions) 95% of observations fall between plus and minus two standard deviations from the mean.

Well… That was not completely exact…

I am sorry.

68, 95, 99 Rule, Rounded

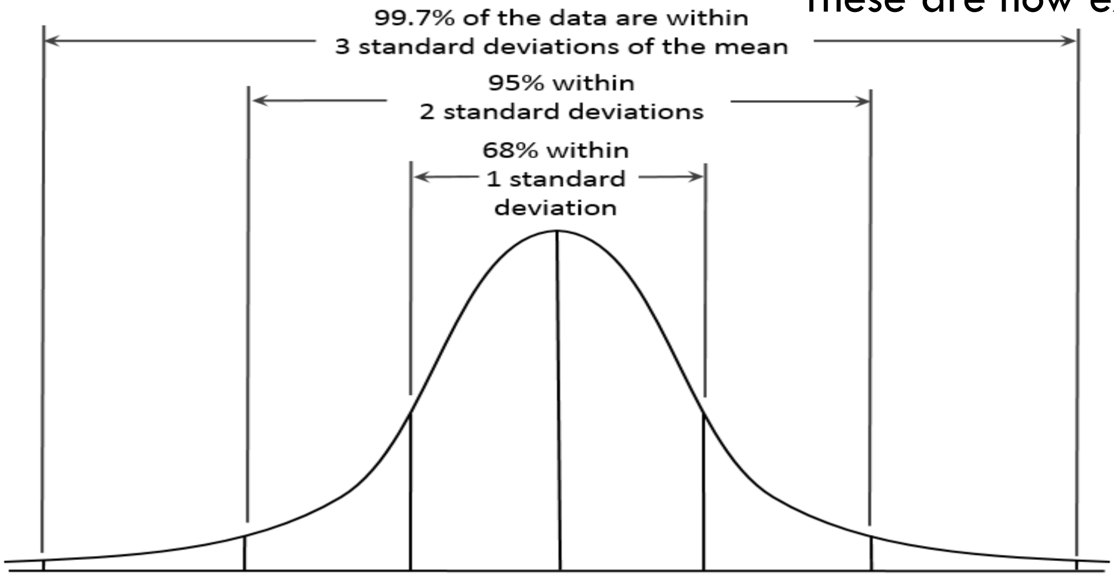
These are rounded approximations.



For normally distributed data: 68% of observations fall between ± 1 SDs
95% of observations fall between ± 2 SDs
99.7% of observations fall between ± 3 SDs

90, 95, 99 Rule, <u>Exact</u>

These are now exact, not rounded.

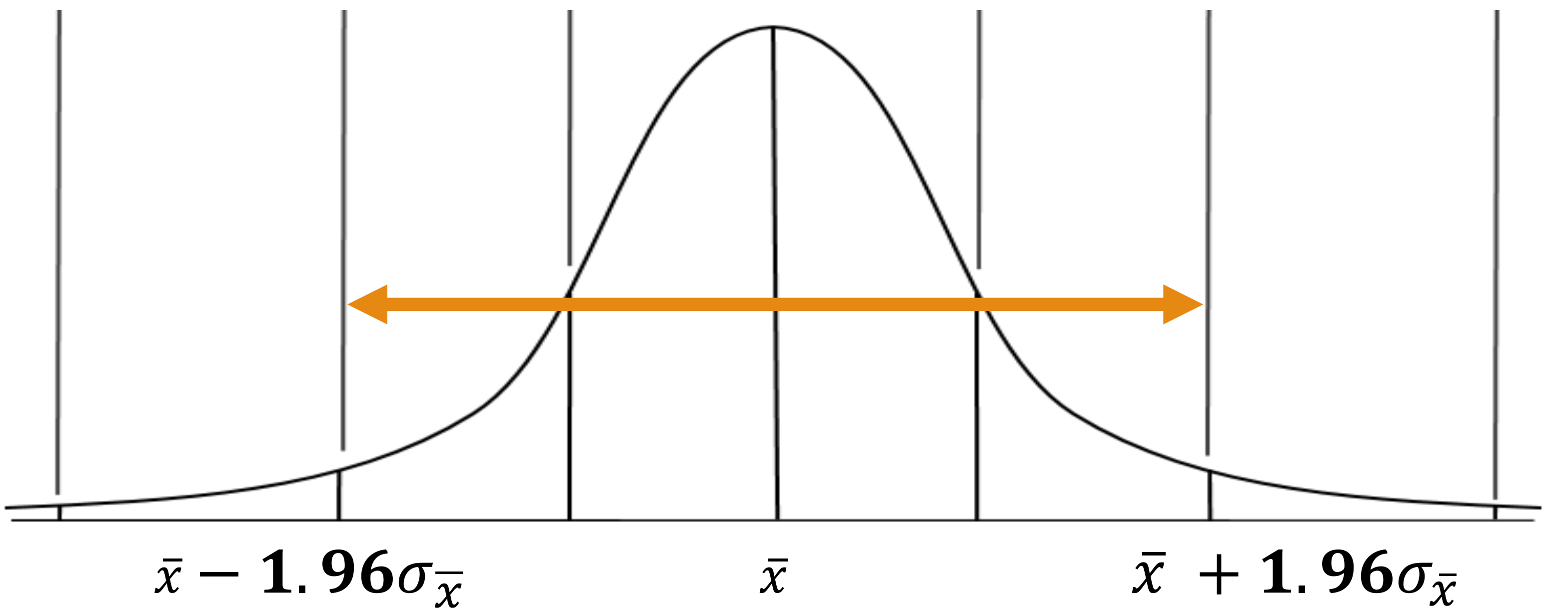


We don't care as much about 68% in this context.

90% of observations fall between ±1.65 SDs
95% of observations fall between ±1.96 SDs
99.7% of observations fall between ±2.58 SDs

Good idea to try to remember these…

95% Confidence Interval and 1.96



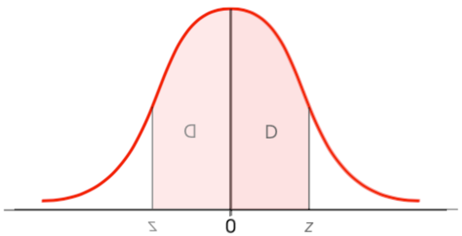
The confidence level of 95% is a fairly common standard in academia (for better or for worse). For much of the rest of the course we will be using the standardized values associated with 95% confidence. When we are dealing with data where the standard deviation is known (σ) we use the standardized z-value of 1.96. In the future, when σ is unknown, we will use different standardized values…

I am sorry, again.

In Case You're Wondering

- In case you're wondering where 1.96 came from, it is from the unit normal table for z-scores. If you look for the z-score that is associated with 95%, you will find this:

.4750 + .4750 = .950



(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
1.96	.9750	.0250	.4750
1.97	.9756	.0244	.4756
1.98	.9761	.0239	.4761
1.99	.9767	.0233	.4767

But how do we know?

Since we will never actually see the sampling distribution (remember we only ever take one sample in the real world) you might be wondering, how do we know if our sample mean is one of the sample means that is too high or too low?

Can we ever know this?

Hm… It all seem so uncertain…

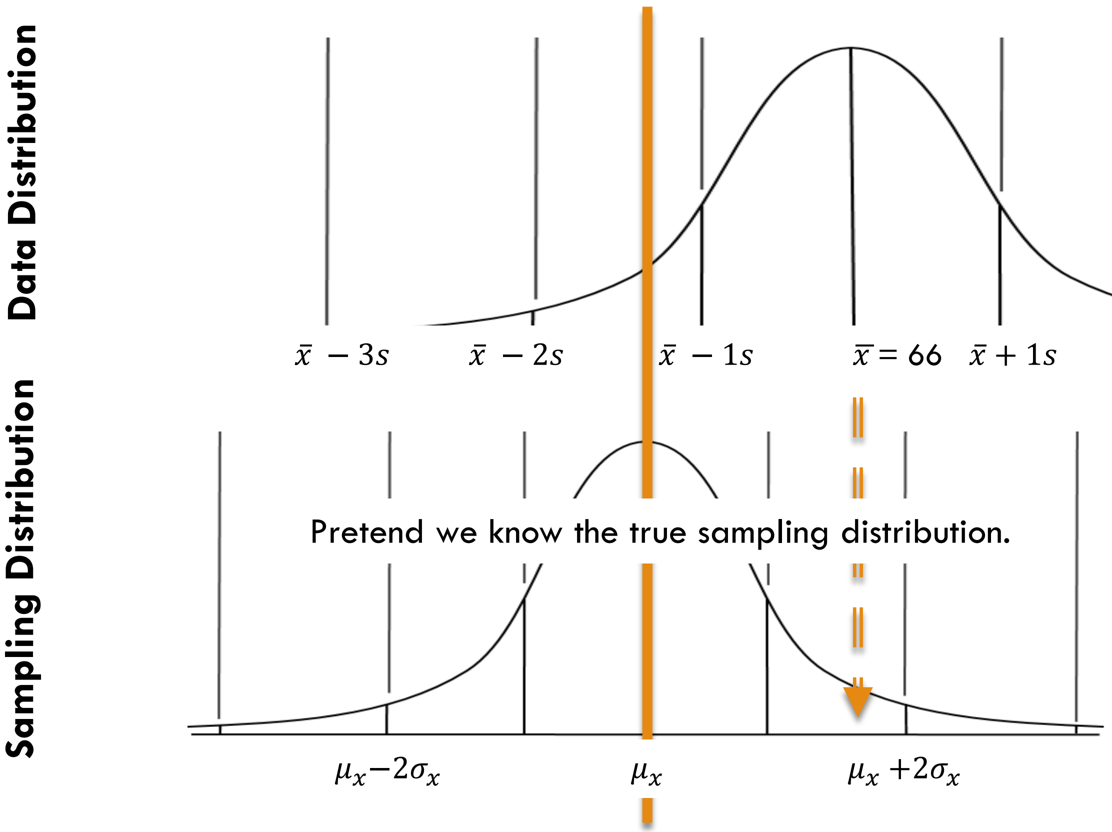
It is… but we can account for some of that uncertainty…

An Ideal World of Knowns

Where we already know the truth…

No, it's not realistic, but it will help us transition to the hard reality of uncertainty.

Height Example With Known Info



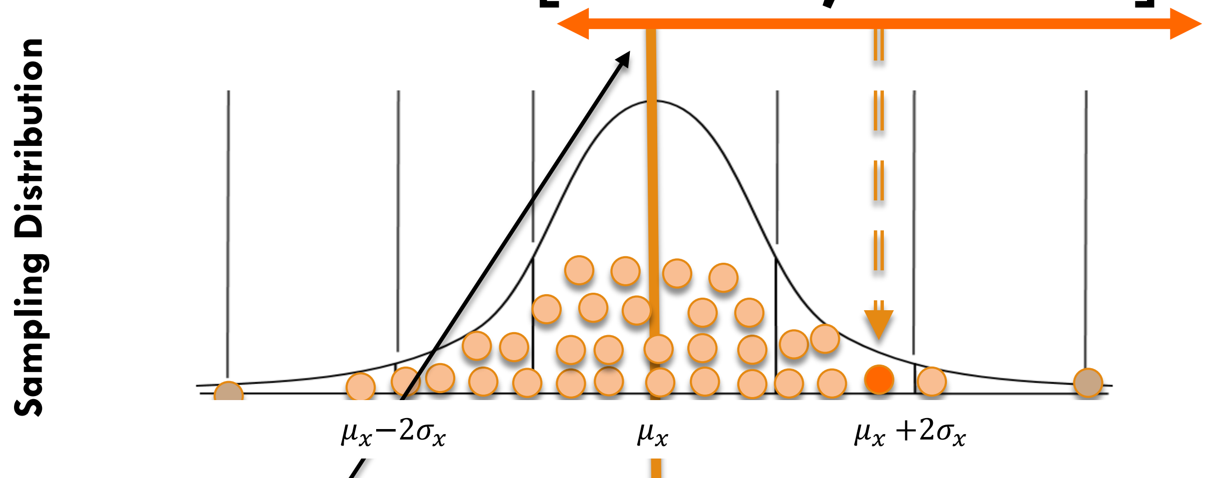
$$\mu = 65''$$
$$\sigma = 3.5''$$

I go out and take a sample of N = 36 women and ask their heights. I calculate an average of $\bar{x} = 66$''. Here, we know the truth is actually 65'', so our sample is actually a little high… But, let's create a confidence interval around our estimate ($\bar{x} = 66$'') and see if we can capture the truth.

Height Example With Known Info

For a sample with mean a of 66" and sample size of 36, using the known standard deviation $\sigma = 3.5''$ we can calculate a 95% confidence interval which is:

[64.86, 67.14]



Did we capture the truth in our Confidence Interval around our sample mean?

Notice how our interval crosses and captures the true μ = 65"

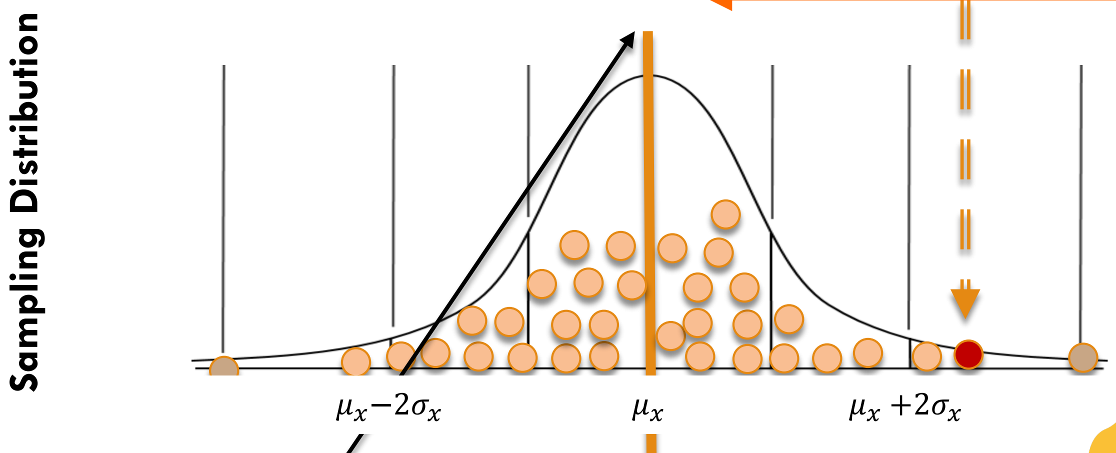
$$\mu = 65''$$
$$\sigma = 3.5''$$

Yes! Hooray! 65" is inside the interval [64.86-67.14]

Height With Known Info Poor Sample

I'm feeling a bit lazy and while I am at a sports conference for volleyball and basketball, I ask 9 women how tall they are. I calculate a sample mean of 68" and create a 95% confidence interval around this which comes out to:

$$[65.7, 70.29]$$



Did we capture the truth in our Confidence Interval around our sample mean?

Notice how this time we our interval does NOT cross and captures the true $\mu = 65$"

$$\mu = 65''$$
$$\sigma = 3.5''$$

No… Our sample missed the truth…

Accepting Uncertainty

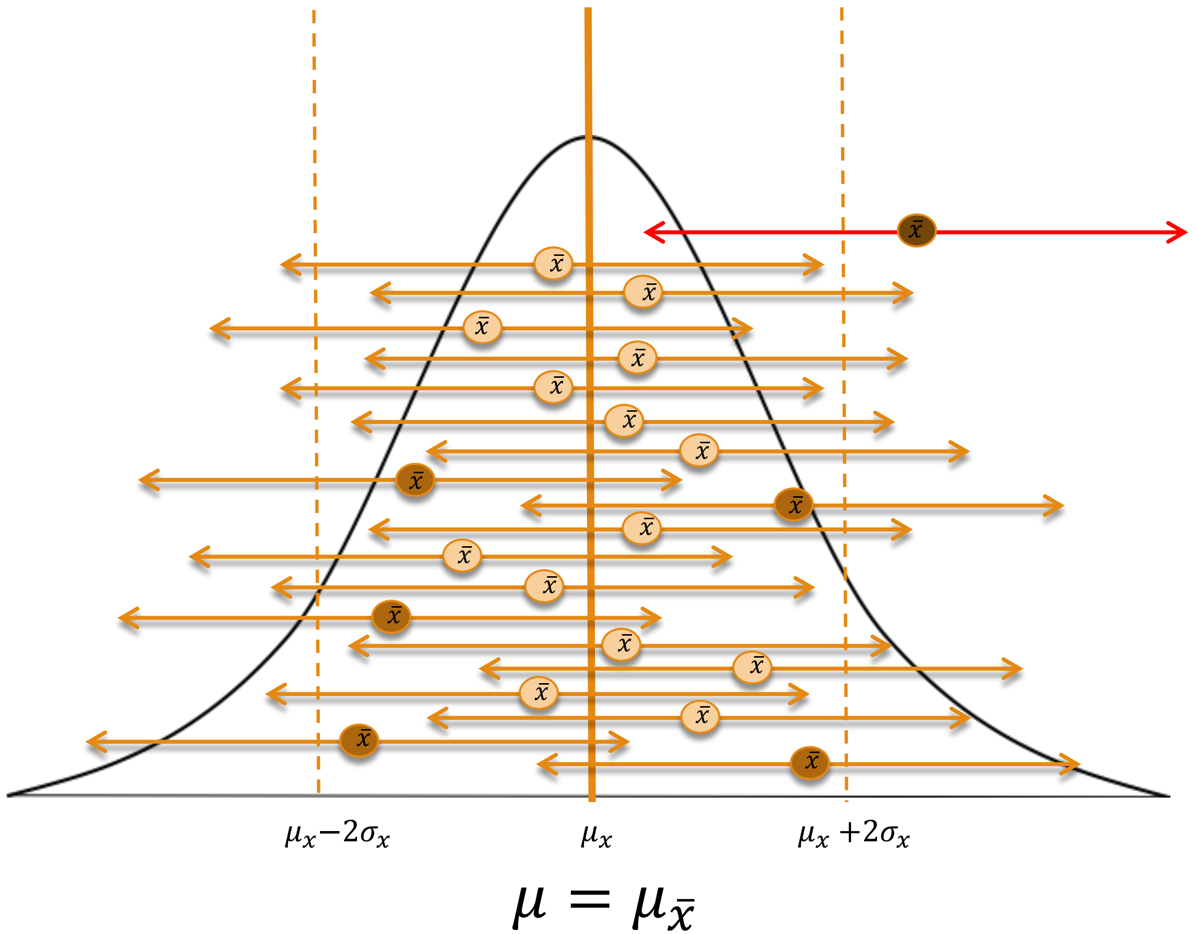
- "How did we miss the truth if we created our 95% confidence interval?? Aren't we 95% confident?!"
  - Yes, indeed, we are 95% confident, but that still leaves 5% we are not so confident about
  - That is a 1 in 20 chance of missing the truth
    - Basically being wrong…
- You might be that 1 in 20, that 5%...

Visualizing the 5%, 1 in 20

Each dot represents a sample point estimate, one sample mean $\bar{x}$

Each arrow, represents the confidence interval associated with that estimate $\bar{x}$

The red arrow, with the dark dot represents when we miss the true mean, that 1 in 20 (95% confidence) chance we miss the actual parameter value μ

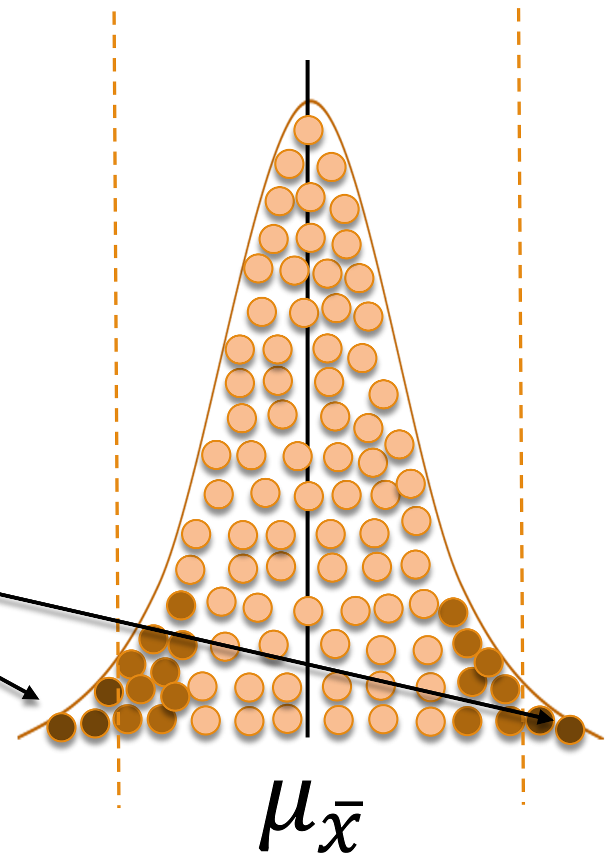


Grab an Estimate

- = $\bar{x}$ within 1.65 σ from true μ
- = $\bar{x}$ within 1.96 σ from true μ
- = $\bar{x}$ within 2.58 σ from true μ

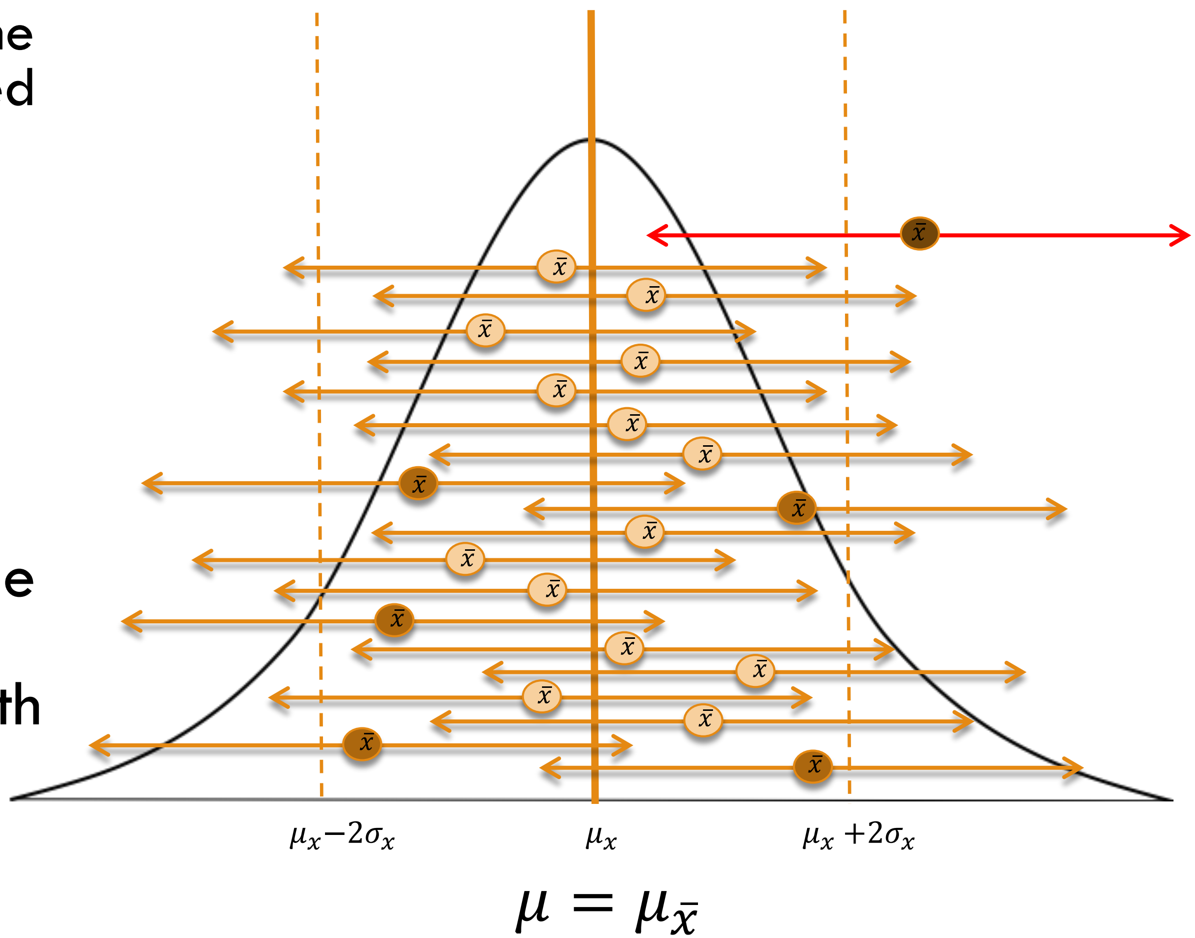
- If I reach into a bag full of 100 point estimates, $\bar{x}$, 5 out of 100 times, I will grab an estimate that does not catch the truth
- In these cases I will "miss" the mark and my CI will not capture the true population parameter, μ

These are misses.



How Confidence Intervals Help Us

- Why do we create confidence intervals?
  - To provide us with the wiggle room we need to account for uncertainty and to give us a better chance of capturing the truth.
- Notice how not a single sample mean actually is exactly the truth, but 95% of them capture the truth in their confidence interval!



Interactive Tool

- Here is a link to an awesome interactive tool by Kristoffer Magnusson (who is awesome). I encourage you to play with it a but on your own to see how different factors affect confidence intervals.

Interactive Visualization of Confidence Intervals

Next Up…

Enough with the theoretical…

Let's actually calculate some stuff!